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Simulation of Magnetoblast Wave over A sphere in a conducting Plasma

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Abstract. *A similarity method is used to study the formation of blast wave in the presence of magnetic field in a one dimensional steady of supersonic flow of a heated gas past a point symmetric bodies . The influence of both transverse and azimuthal magnetic field Mach number without radiation on the wave front are examined.*

INTRODUCTION :

The study of blast wave propagation, over an obstacle is largely motivated by the need of predict the dynamic response of structures to an internal or external explosions . As well as, by need to further understand complex of non stationary gas dynamic phenomenon.

In the past, experimental , analytical and numerical results of interaction, of planar shock waves with cylinders, have been documented by several authors. Witham [1] has developed an approximate theory for the dynamics of shock waves. Bryson and Gross [2] extended witham's theory to two and three dimensional bodies such as cylinder and sphere. Pai [3] Green span [4], Christer and Helliwell [5] have studied a cylindrical shock wave of line explosions, produced on account of sudden release of a finite amount of energy expanding outwards in a conducting plasma to a magnetic field. Dev Ray [6] and Vishwakarma [7] have obtained similarity solution for strong cylindrical blast wave in a conducting non uniform medium. Recently Vishwakarma and Yadav found solutions for blast wave with frozen in magnetic field [8].

The present paper deals with numerical simulation of spherical blast wave obtained when a large amount of energy is suddenly released in a relatively small region. A disturbance headed by a strong shock wave called blast wave is produced and propagates into the surrounding gaseous medium subjected to magnetic field. Here we consider both transverse and azimuthal magnetic field. This blast wave propagates in the medium whose density varies with time and total energy of the wave remains constant. Equations of motions are solved by similarity method. Analytical results are obtained for flow variables and comparisons are obtained for both azimuthal and transverse magnetic field.

EQUATION OF MOTION :

The equations of motion for one dimensional unsteady adiabatic flow of a perfect gas with transverse magnetic field are :

$$\frac{\partial}{\partial t}(\log \rho) + u\frac{\partial}{\partial r}(\log \rho) + \frac{\partial u}{\partial r} + \frac{2u}{r} = 0 \quad , \qquad (1)$$

$$\frac{\partial}{\partial t}(\log H_\theta) + u\frac{\partial}{\partial r}(\log H_\theta) + \frac{\partial u}{\partial r} + \frac{u}{r} = 0 \quad , \qquad (2)$$

$$\frac{\partial}{\partial t}(\log H_z) + u\frac{\partial}{\partial r}(\log H_z) + \frac{\partial u}{\partial r} + \frac{u}{r} = 0 \quad , \qquad (3)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{1}{2\rho}\frac{\partial}{\partial r}(H_\theta^2 + H_z^2)$$

$$+ \frac{1}{r\rho} + (H_\theta^2 + H_z^2) = 0 \quad , \qquad (4)$$

$$\frac{\partial}{\partial t}(\log p\rho^{-r}) + u\frac{\partial}{\partial r}(\log p\rho^{-r}) \quad ; \qquad (5)$$

the magnetic permeability is taken to be unity. Other symbols have their usual meanings and they are functions of r and t only γ is the specific heat ratio.

BOUNDARY CONDITION

Following Whitham [1], strong shock boundary conditions are

$$\rho_1 = \frac{\gamma + 1}{\gamma - 1} \rho_0 \ , \tag{6}$$

$$p_1 = \frac{2}{\gamma + 1} p_0 \dot{R}^2 , \tag{7}$$

$$u_1 = \frac{2}{\gamma + 1} \dot{R} \ , \tag{8}$$

$$H_\theta = \frac{\gamma + 1}{\gamma - 1} H_{\theta_0} , \tag{9}$$

$$H_z = \frac{\gamma + 1}{\gamma + 1} H_{Q_0} \ ; \tag{10}$$

SIMILARITY SOLUTIONS

Let the similarity variable ξ be in the form

$$\xi = \frac{r}{R} \ , \tag{11}$$

$$R = R\,[t] \ ;$$

We seek the solution for partial differential equation (1) - (5) in the form

$$
\begin{aligned}
u &= \dot{R}\, v(\xi) \ , \\
\rho &= \rho_0\, g(\xi) \ , \\
p &= p_0\, \dot{R}^2\, f(\xi) \ , \\
H_\theta &= \rho_0^{1/2}\, \dot{R}\, h(\xi) \ , \\
H_z &= \rho_0^{1/2}\, \dot{R}\, \bar{h}(\xi) \ ;
\end{aligned}
\tag{12}
$$

where v, g, f, h and $\bar{h}$ are function of ξ. The scales R, ρ_0 and $\dot{R}$ are time dependent in some manner as unknown.

SOLUTION OF EQUATION OF MOTION:

We now substitute the equations [12] into equation [1] – [5] taking account of the definition of similarity variable [11]

$$\frac{\dot{\rho}}{\rho_0} + \frac{\dot{R}}{R}\left[v'(\xi) + \frac{g'(\xi)}{g(\xi)}(v-\xi) + \frac{2v(\xi)}{\xi}\right] = 0 \quad , \qquad (13)$$

$$\frac{R}{\dot{R}}\left[\frac{\ddot{R}}{\dot{R}}\right] + \frac{\dot{\rho}}{2\rho_0}\left\{\right\} + [v(\xi)-\xi]\frac{h'(\xi)}{h(\xi)} + v'(\xi) + \frac{v(\xi)\,h(\xi)}{\xi} \quad , \qquad (14)$$

$$\frac{R}{\dot{R}}\left[\frac{\ddot{R}}{\dot{R}}\right] + \frac{\dot{\rho}}{2\rho_0}\left\{\right\} + [v(\xi)-\xi]\frac{\bar{h}'(\xi)}{\bar{h}\xi} + v'(\xi) + \frac{v(\xi)\,\bar{h}(\xi)}{\xi} = 0 \quad , \qquad (15)$$

$$\frac{R\ddot{R}}{\dot{R}}v(\xi) + v'(\xi)[v(\xi)-\xi] + \frac{1}{g(\xi)}\left[f'(\xi)+h(\xi)\left(h'(\xi)+\frac{h(\xi)}{\xi}\right)\right]$$

$$+\bar{h}(\xi)\left[\bar{h}'(\xi)+\frac{\bar{h}}{\xi}\right] = 0 \quad . \qquad (16)$$

$$\frac{R}{\dot{R}}\frac{d}{dt}\left(\log \rho_0^{1-\gamma}\dot{R}^2\right) + [v(\xi)-\xi]\left\{\frac{f'(\xi)}{f(\xi)} - \frac{\gamma g'(\xi)}{g(\xi)}\right\} = 0 \quad . \qquad (17)$$

In order that similarity solution (12) be meaningful, it is necessary that variable t and ξ in equation (13), (14), (15), (16) and (17) must be separable. This is possible if $\frac{R\ddot{R}}{\dot{R}^2}$ = constant and $\frac{\dot{\rho}_2}{\rho_0}$ = constant, $\frac{\dot{R}}{R}$ = constant in (13) which yields

$$R = A t^{\alpha} \quad , \tag{18}$$

and

$$\rho_0 = B t^{\beta} \quad . \tag{19}$$

Put these values in equation (13) – (17)

$$\frac{\beta}{\alpha} + v'(\xi) + \frac{g'(\xi)}{g(\xi)}[v(\xi)-\xi] + \frac{2v(\xi)}{\xi} = 0 \quad , \tag{20}$$

$$\frac{2(\alpha-1)+\beta}{2\alpha} + (v(\xi)-\xi)\frac{h'(\xi)}{h(\xi)} + v'(\xi) + \frac{v(\xi)h(\xi)}{\xi} = 0 \quad . \tag{21}$$

$$\frac{2(\alpha-1)+\beta}{2\alpha} + [v(\xi)-\xi]\frac{\bar{h}'(\xi)}{\bar{h}(\xi)} + v'(\xi) + \frac{v(\xi)\bar{h}(\xi)}{\xi} = 0 \quad . \tag{22}$$

$$\left(\frac{\alpha-1}{\alpha}\right) v(\xi) + v'(\xi)[v(\xi)-\xi] + \frac{1}{g(\xi)}\left\{f'(\xi) + h(\xi)\left(h'(\xi) + \frac{h(\xi)}{\xi}\right)\right.$$
$$\left. + \bar{h}(\xi)\left(\bar{h}'(\xi) + \frac{\bar{h}(\xi)}{\xi}\right)\right\} = 0 \quad , \tag{23}$$

$$\frac{2(\alpha-1)+(1-\gamma)\beta}{\alpha} + [v-(\xi)-\xi]\left(\frac{f'(\xi)}{f(\xi)} - \frac{\gamma g'(\xi)}{g(\xi)}\right) = 0 \quad . \tag{24}$$

Now we consider the total energy E of the flow as

$$E = 2\pi \int_0^R \left[\frac{1}{2}\rho u^2 + \frac{P}{\gamma - 1} + \frac{1}{2}(H_\theta^2 + H_z^2) \right] r\,dr \quad , \tag{25}$$

$$E = 2\pi \int_0^1 \left[\frac{1}{2}\rho u^2 + \frac{P}{\gamma - 1} + \frac{1}{2}(H_\theta^2 + H_z^2) \right] \xi A^2 t^{2\alpha}\, d\xi \quad ,$$

substitute the values of ρ, u, P, H_θ and H_z from eqn [12], we get

$$E = 2\pi\, {}_0\dot{R}^2 \int_{\xi_0}^1 \left[\frac{1}{2} g(\xi)\, v^2(\xi) + \frac{f(\xi)}{\gamma - 1} + \frac{1}{2}\left(h^2(\xi) + \bar{h}^2(\xi)\right) \right] \xi A^2 t^{2\alpha}\, d\xi$$

$$= 2\pi B A^4 \alpha^2 t^{4\alpha+\beta-2} \int_{\xi_0}^1 \left[\frac{1}{2} g(\xi)\, v^2(\xi) + \frac{f(\xi)}{\gamma - 1} + \frac{\left(h^2(\xi) + \bar{h}^2(\xi)\right)}{2} \right] \xi\, d\xi \quad . \tag{26}$$

RESULT AND DISCUSSIONS

In this problem we have calculated our results in the form

$$\frac{u}{u_1} = \frac{\gamma + 1}{2}\, u(\xi) \quad , \tag{27}$$

$$\frac{\rho}{\rho_1} = \frac{\gamma - 1}{\gamma + 1}\, g(\xi) \quad , \tag{28}$$

$$\frac{p}{p_1} = \frac{\gamma + 1}{2}\, f(\xi) \quad , \tag{29}$$

$$\frac{H_\theta}{H_{\theta 1}} = \frac{\gamma - 1}{\gamma + 1}\, M_A\, h(\xi) \quad , \tag{30}$$

$$\frac{H_z}{H_{z1}} = \frac{\gamma - 1}{\gamma + 1}\, M_A\, \bar{h}(\xi) \quad ; \tag{31}$$

where M_A is the alfven Mach number.

Equations (24-28) have been integrated by software Matlab for $\gamma = 1.4$ □=0.25, 0.50, and M_A = 10, 100 The variation of flow variables are illustrated through graphs.

In this problem for the shock wave which is produced by a point explosion moves in conducting gas, we have discussed two cases and make a comparison between them.

Case-I Propagation of spherical blast wave with azimuthal magnetic field.

Care-II Propagation of spherical blast wave with transverse magnetic field.

It is observed that when the shock radius is contracting that is $\alpha = 0.25$, there is sharp increase in velocity density , pressure and magnetic field of the fluid particles behind the shock surface as compare $\alpha = .50$ when the contraction in the shock radius is less.

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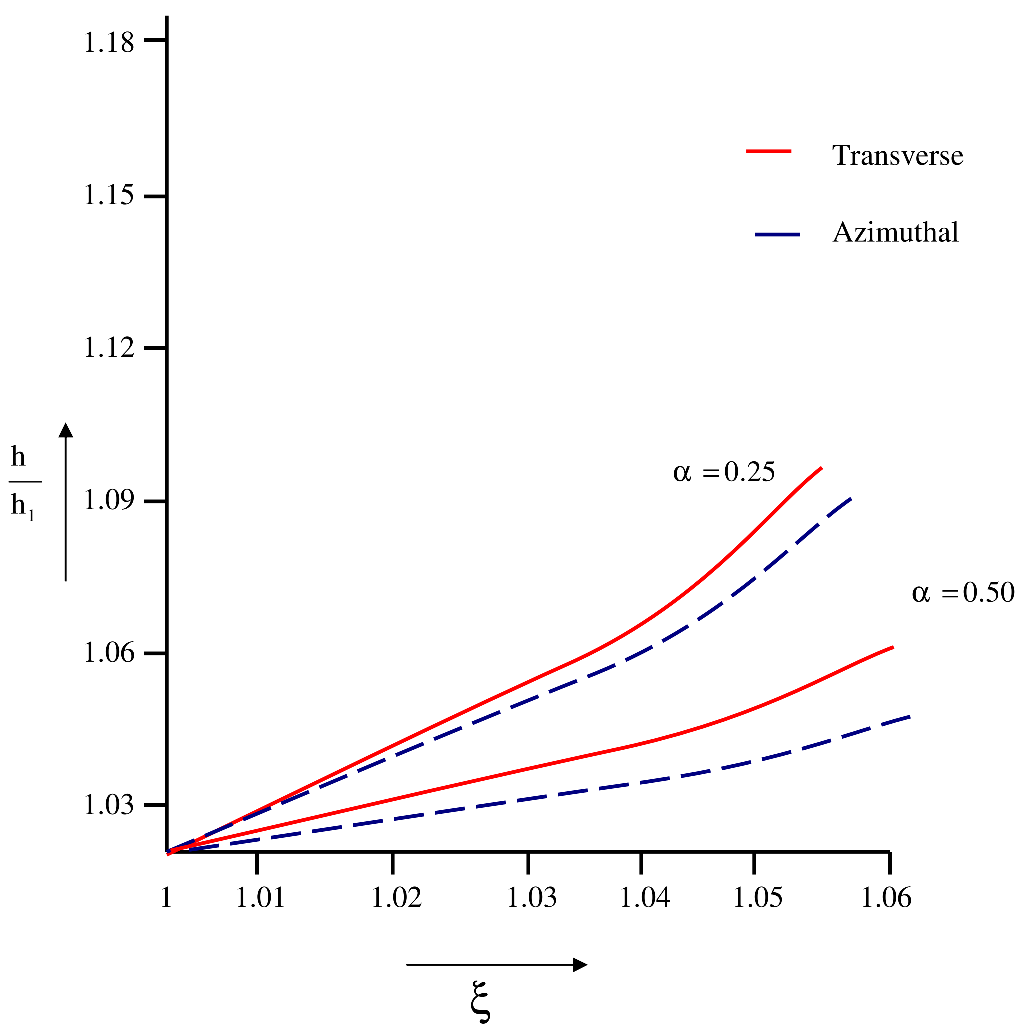
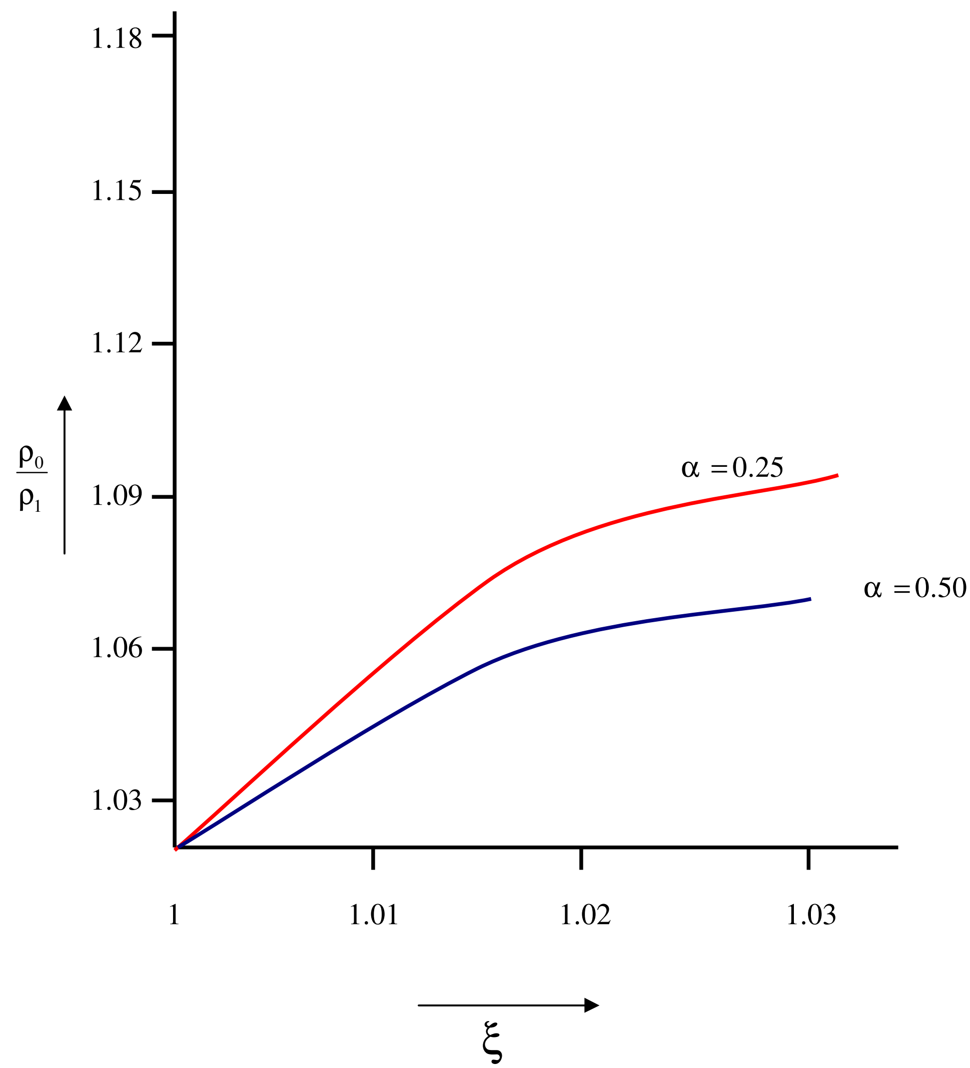
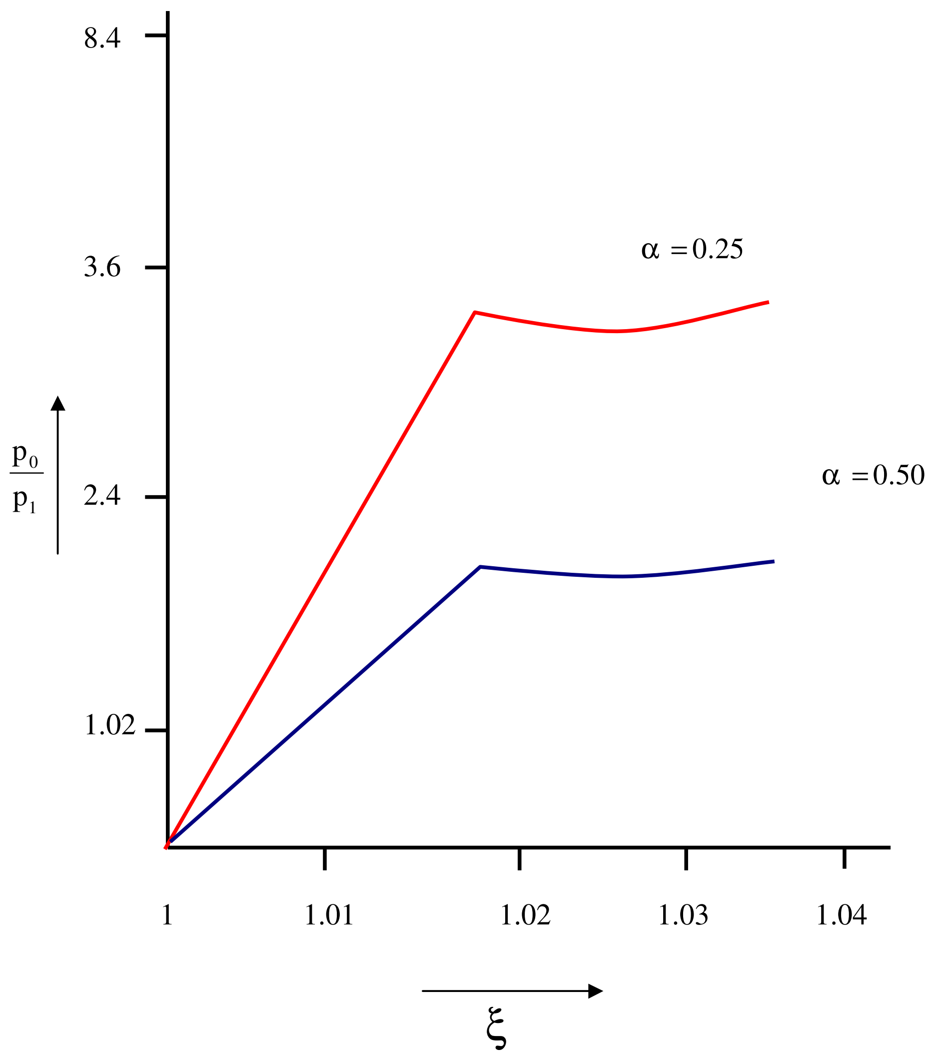


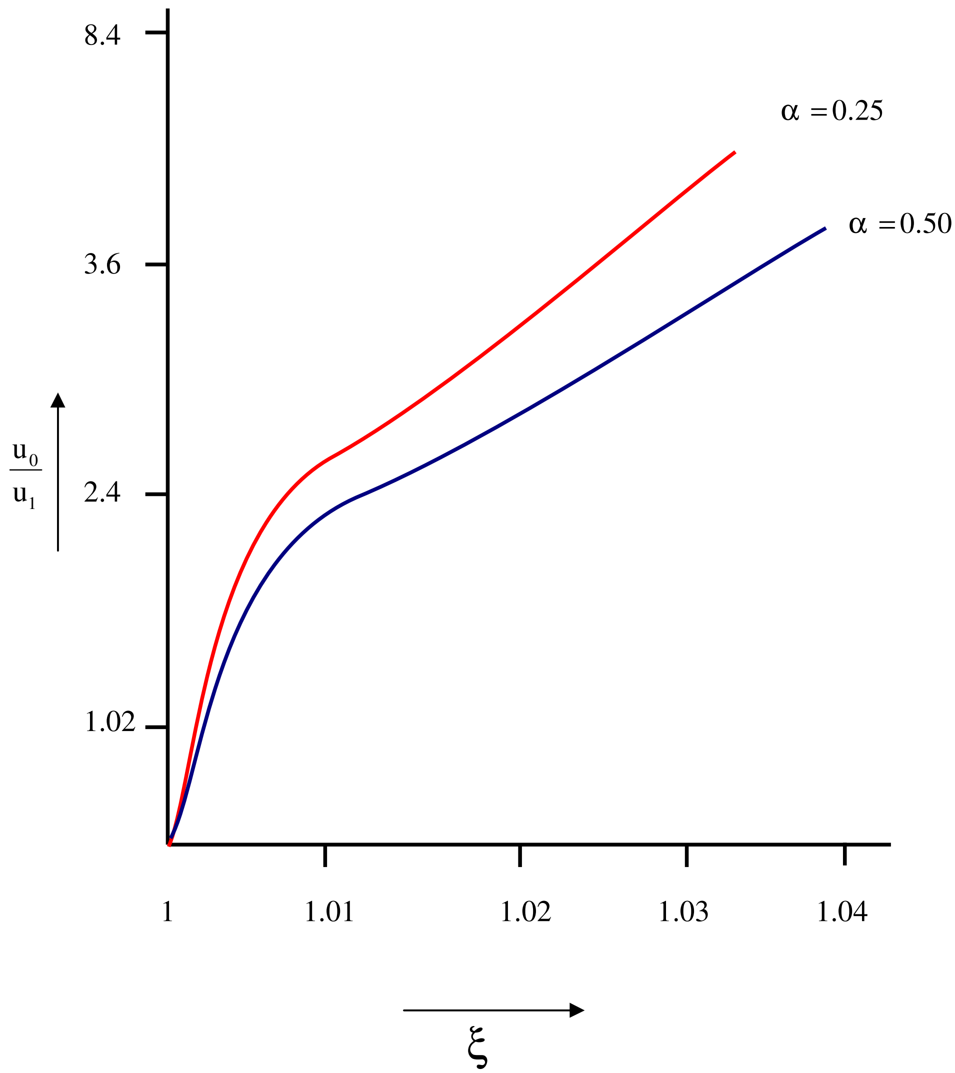
Magnetic field distribution



Density Distribution



Pressure Distribution



Velocity Distribution