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Similarity Solution of Isothermal Shock with Radiative Heat Flux

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Abstract. *In this paper the self similar model of isothermal cylindrical shock wave has studied, taking magnetic radiative heat flux into account where total energy of the wave is variable and atmosphere is uniform.*

INTRODUCTION :

The problems of radiative heat transfer in fluids have their importance in recent years due to the increasing speeds of bodies through the atmosphere and the very high temperatures attained by gases in motion for example, effects of radiations are of significance in the field of nuclear power and space research. Elliot [1], Wang [2] and Helliwell [3] have considered the problem of shock with thermal radiation using similarity method. Sachdeva and Bhatnagar [4] has studied the propagation of an isothermal shock in stellar medium. Singh [5] has discussed the self similar problems of isothermal shock with radiative heat flux .Sachdeva [6] has considered the shock with all radiation parameters of radiation in magneto gas dynamics.

In this paper we have consider propagation of cylindrical shock wave under isothermal condition. In the isothermal condition temperature gradient becomes zero behind the shock and radiation effects are already implicitly involved. Here radiation pressure and energy have been neglected only radiation flux has been taken into account. We have also assumed the gas to be gray and opaque. We have ignored radiation flux in front of the shock in comparison to behind it.

EQUATION OF MOTION :

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial t} + \rho\frac{\partial u}{\partial r} + 2\rho\frac{u}{r} = 0 \quad , \qquad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \quad , \qquad (2)$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} + \frac{\gamma p}{\rho}\left[\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right] + \frac{(\gamma - 1)}{\gamma 2}\frac{\partial}{\partial r}(r^2 F) = 0 \quad , \qquad (3)$$

where u, p, ρ and F are the velocity, pressure density and radiative flux at a radial distance r from the centre of explosion at time t.

Equation of state for ideal gas is

$$e = \frac{p}{(\gamma - 1)\rho} \quad \text{and} \quad P = \Gamma \rho T \quad . \qquad (4)$$

Following Helliwell, the variation in the radiative flux is given by

$$\frac{\partial F}{\partial r} + \frac{F}{r} = 4\pi k B \quad , \qquad (5a)$$

where B denotes plank's radiation function and is given by $B = \sigma T^4 / \pi, \sigma$ the stefan's constant and K is the local volumetric absorption coefficient given as

$$k = k\rho^{\alpha} T^{\beta} \qquad (5b)$$

where k, □□ are constant.

SHOCK CONDITIONS :

The disturbance is headed by isothermal shock and the jump conditions are

$$\rho_1 (u - u_1) = \rho_0 U = m \quad , \qquad (6)$$

$$P_1 - P_0 = m_s u_1 \quad , \qquad (7)$$

$$e_1 + \frac{P_1}{\rho_1} + \frac{1}{2}(U - u_1)^2 - \frac{F}{m_s} = e_0 + \frac{P_0}{\rho_0} + \frac{1}{2}U^2 \quad , \tag{8}$$

$$T_1 = T_0 \quad , \tag{9}$$

where subscripts 1 and 0 for the regions just behind and just ahead of the shock surface respectively and U denotes the shock velocity. After some manipulation the above equation [6] to [9] reduce to

$$U_1 = a_0 \left[M - \frac{1}{\gamma M} \right] \quad , \tag{10}$$

$$\rho_1 = \rho_0 \; [\gamma M^2] \quad , \tag{11}$$

$$P_1 = P_0 \; \gamma M^2 \quad , \tag{12}$$

$$F_1 = \rho_0 \, a_0^{1/2} \left[\frac{1}{\gamma M^2} - M^3 \right] \quad , \tag{13}$$

where $a_0 = \left[\frac{\gamma P_0}{\rho_0} \right]^{1/2}$ is the sound velocity along the positive characteristic

$$\frac{dr}{dt} = u + a \quad . \tag{14}$$

We have following characteristic equations , by CCW method [7,8,9]

$$dp + \rho a \, du + \left[\rho a^2 \frac{u}{r} + \frac{\gamma - 1}{r\,j} \frac{\partial}{\partial r}(Fj) \right] \frac{dr}{u + a} = 0 \quad . \tag{15}$$

ANALYSIS:

[a] Uniform atmosphere

When atmosphere is uniform

we take $\rho_0 = P_0 = \text{constant}$.

Substituting the equation [5a] ,[5b] and shock conditions [10] to [13] into the equation [15] we get

$$(\gamma M^2 + 2M + 1)\ \frac{dM}{dR} + (\gamma\ M^2\ (\gamma M^2 - 1)\ \frac{1}{\gamma M^2} + \ (\gamma M - 1)\frac{1}{R}$$

$$+ N\ \gamma^{\alpha-\beta-3}\ M^{2\alpha+1}\ \rho_0^{\alpha-1\ 2\beta+5}\ \frac{a_0}{\gamma M^2} + \gamma\ M - 1 = 0 \quad (16)$$

,

where R is shock radius and

$N\ =\ 4\ (\gamma - 1)\ \ \sigma k\ /\ r^{\beta+4}$,

if we take $\alpha = 1$ and $\beta = -\ 2.5$ then equation [16] becomes

$$\gamma^2 M\ +\ \gamma\ (\gamma + 2)\ +\ \frac{2\gamma}{M}\ +\ (\gamma - 2)/M^2\ -\ \frac{1}{M^3}]\ \frac{dM}{dR}$$

$$+\ \gamma\ \frac{(\gamma\ M^2 - 1)}{MR}\ +\ \sqrt{\gamma}\ \ N\ = 0 \quad (17)$$

[b] <u>Non uniform atmosphere</u>

In this case, we have assumed the density distribution

$$\rho_0\ = \rho_e\ r^{-\delta} \quad (18)$$

and pressure distribution

$$P_0\ = P_c\ \bar{r}^{\mu} \quad (19)$$

where ρ_c , P_c , δ and μ are constants.

Now equation (15) after simplification according to (17) yields

$$\left(\gamma M^2 + 2M + 1\right)\frac{dM}{dR} + \{[(\delta - \mu)\ (\gamma M^3 - M) - 2\mu\, M^2\,]/2\}$$

$$+\ \{\gamma M^2\ (\gamma M^2 - 1)/\{\gamma M^2 + \gamma M - 1\}\ \frac{1}{2}\ \ +\ \ \frac{\sqrt{\gamma}\ NM^3}{\gamma\ M^2 + \gamma M - 1}\ \ =\ 0 \quad (20)$$

RESULTS AND DISCUSSIONS :

The equation (17) and (20) have been numerically integrated assuming initially M = 4 at R = 0.01. The solution will be obtained in the form

[A] Uniform atmosphere

$$\frac{U}{a_0} = \frac{M-1}{\gamma M} \quad , \tag{21}$$

$$\frac{\gamma}{\rho_0} = \gamma M^2 \quad , \tag{22}$$

$$\frac{p}{p_0} = \gamma M^2 \quad , \tag{23}$$

$$\frac{F}{\rho_0 a_0} = \frac{1}{2}\left\{\frac{1}{\gamma}M - M^3\right\} \quad . \tag{24}$$

As R increases, the Mach number M decreases in each cases. The tables (A) shows the nature of flow variables in uniform atmosphere and table (B) reveal the nature of flow variables in non uniform atmosphere from these tables we can compare our results for cylindrical shock. Other details about increasing and decreasing nature of flow variables can easily be studied through tables. From tables we observes that our results agrees with the result given by Singh [10,11] and Vishwarkarma [12]

Table-A

N= 3.5

R	M	U/a_0	ρ / ρ_0	P/P_0	$F / \rho_0 a_0$
0.01	0.40000	3.6916	18.1000	16.2001	-31.7958
0.16	3.7575	3.6495	18.0300	15.5734	-30.6228
0.31	3.7182	3.6003	17.8191	15.2321	-30.4132
0.46	3.6852	3.5921	17.6571	15.0922	-30.0210
0.61	3.4751	3.5159	17.4852	14.8772	-29.779
0.76	3.25280	3.4301	16.9002	14.0835	-26.3150
0.91	3.0822	3.3925	16.5249	13.8702	-25.8752
1.06	2.9722	3.1507	15.9852	13.3551	-25.2015
1.31	2.7046	2.8723	15.5084	12.9512	-24.8015
1.51	2.3253	2.5679	15.1731	12.6285	-24.5321

Table-B

N= 3.5

R	M	U/a_0	ρ / ρ_0	P/P_0	$F / \rho_0 a_0$
0.01	4.0000	1.1230	181.0000	1512.0000	-10167-3486
0.16	8.1906	4.4657	223.3095	492.8240	-3296.7482
0.31	9.6389	7.3501	238.9689	346.2609	-2513.5920
0.46	11.2707	8.1892	259.7560	306.7064	-2261.9016
0.61	12.7925	9.9512	275.7575	296.9942	-2019.2092
0.76	13.6215	10.9752	288.2051	284.3882	-1850.6912
0.91	14.1235	12.9894	299.1523	261.2074	-1739.591
1.06	15.7932	14.3523	302.3599	257.9215	-1668.2265

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